Normalised solutions to a fractional Schrödinger equation in the strongly sublinear regime

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Schrödinger-type equations model a lot of natural phenomena and their solutions have interesting and important properties: one of them is the conservation of mass, which gives rise to the search for *normalised solutions*. In this talk, I will explain a possible approach to solve

$$\begin{cases} (-\Delta)^s u + \mu u = g(u), \\ \int_{\mathbb{R}^N} u^2 \,\mathrm{d}x = m, \\ (\mu, u) \in \mathbb{R} \times H^s(\mathbb{R}^N), \end{cases}$$

where $N \geq 2$, $0 < s < 1$, and $m > 0$ is prescribed, in cases that include the so-called *strongly sublinear regime*:

$$\lim_{t \to 0} \frac{g(t)}{t} = -\infty. \tag{1}$$

This makes a direct variational approach impossible because the energy functional is not well-defined in $H^s(\mathbb{R}^N)$. In the proposed approach, when m is sufficiently large, a family of approximating problems is considered so that the energy functional is of class $\mathcal{C}^1$ and a corresponding family of solutions is obtained, which eventually converge to a solution to the original problem. When (1) holds, the previous result for a suitably translated problem is exploited to obtain a solution for any $m > 0$.

This is joint work with Marco Gallo (Catholic University of the Sacred Heart, Brescia, Italy).