Stability of the Möbius group and simple bubbles of the (2-dim) H-system

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We present quantitative stability aspects of the class of Möbius transformations of $\mathbb{S}^{n-1}$ among maps in the critical Sobolev space $W^{1,n-1}$. The case of $\mathbb{S}^{n-1}$- and general $\mathbb{R}^n$-valued maps will be addressed. In the latter, more flexible setting, unlike similar in flavour results for maps defined on domains of $\mathbb{R}^n$, not only a conformal deficit is necessary, but also a deficit measuring the distortion of $\mathbb{S}^{n-1}$ under the maps in consideration which is introduced as an associated isoperimetric deficit.

Next, we consider conformal transformations of $\mathbb{S}^2$ of arbitrary degree, those being entire solutions $\omega \in \dot{H}^1(\mathbb{R}^2;\mathbb{R}^3)$ of the H-system

$$\Delta\omega = 2\omega_x \wedge \omega_y \ \text{ in } \mathbb{R}^2\,,$$

usually referred to as *bubbles*. Contrary to conjectures raised in the literature, we find that bubbles with degree at least three can be degenerate: the linearized H-system around a bubble can admit solutions that are not tangent to the smooth family of bubbles. We then give a characterization of the degenerate bubbles, and present optimal stability estimates in a neighborhood of any single bubble in the form of Łojasewicz-type inequalities.

The talk will be based on previous works in collaboration with Stephan Luckhaus, Jonas Hirsch, and more recent (also ongoing) ones with Andre Guerra and Xavier Lamy.