Cohomology of polynomial growth, extension of cyclic cocycles and numeric invariants of Dirac operators

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Based on joint work with

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Dirac operator: geometric set-up

- We consider **two** geometric situations
- **(1)**: a smooth compact manifold without boundary M and a Galois coverings $\Gamma \to \widetilde{M} \to M$.
- We then consider $\widetilde{D}$, a Γ-equivariant operator, lift of D on M
- **(2)**: we also consider a semisimple Lie group G and a G-proper manifold X such that X/G is compact
- in this context we consider D_X, an equivariant Dirac operator
- We are interested in K-theory invariants associated to these operators and, above all, to ways to extract **numeric invariants** out of these K-theory invariants.
- We want geometric applications.

Old and new invariants of Dirac operator

- We begin with M and $\Gamma \to \widetilde{M} \to M$.
- One way to organize relavant K-theory invariants associated to D and $\widetilde{D}$ is through the Higson-Roe analytic surgery sequence.
- This allows to recover known invariants, such as
  - the fundamental class $[D] \in K_*(M)$
  - the index class $\mathrm{Ind}_\Gamma(\widetilde{D}) \in K_*(C_r^*\Gamma)$

  but also to define a new invariant:
  - the rho class $\rho(\widetilde{D})$ of an invertible Dirac operator .

The Higson-Roe analytic surgery sequence

• The Higson-Roe analytic surgery sequence is a K-theory sequence that can be written as follows:

$$\cdots \to K_{*+1}(C^*_{red}\Gamma) \to \mathrm{S}^\Gamma_*(\widetilde{M}) \to K_*(M) \to K_*(C^*_{red}\Gamma) \to \cdots$$

• The group $\mathrm{S}^\Gamma_*(\widetilde{M})$ is the analytic surgery group

• $\mathrm{S}^\Gamma_*(\widetilde{M}) := K_{*+1}(D^*(\widetilde{M})^\Gamma)$

• $D^*(\widetilde{M})^\Gamma$ is the norm closure of $D^*_c(\widetilde{M})^\Gamma \subset \mathcal{B}(L^2(\widetilde{M}))$

• $D^*_c(\widetilde{M})^\Gamma$ is the algebra of Γ-equivariant bounded operators on L^2 that are of finite propagation and pseudolocal (i.e. $[f, T]$ is compact for any $f \in C^\infty_c(\widetilde{M})$).

• We are short here because we shall see a different incarnation of this sequence in a moment.

The rho class of an invertible operator

• As anticipated, the group $K_*(C^*_{red}\Gamma)$ is the home of the Index class $\mathrm{Ind}_\Gamma(\widetilde{D})$ and the group $K_*(M)$ is the home of the fundamental class $[D]$

• the analytic surgery group $\mathrm{S}^\Gamma_*(\widetilde{M}) = K_{*+1}(D^*(\widetilde{M})^\Gamma)$ is the home of the rho-class $\rho(\widetilde{D})$ of an L^2-invertible Dirac operator $\widetilde{D}$

• we shall be happy with the case M odd dimensional. Then

$$\rho(\widetilde{D}) := [\Pi_>(\widetilde{D})] \in \mathrm{S}^\Gamma_1(\widetilde{M}) = K_0(D^*(\widetilde{M})^\Gamma)$$

Mapping **geometric** surgery to **analytic** surgery

- why is the Higson-Roe analytic surgery sequence interesting and why is the word **surgery** used ?
- Consider the surgery exact sequence in topology (Browder, Novikov, Sullivan, Wall) associated to M^n orientable:
  $(\star) \qquad L_{n+1}(\mathbb{Z}\Gamma) \dashrightarrow \mathcal{S}(M) \to \mathcal{N}(M) \to L_n(\mathbb{Z}\Gamma)$
- Higson and Roe proved that one can map $(\star)$ to the Higson-Roe analytic surgery sequence (2005)
- consider the Stolz' sequence for positive scalar curvature metrics for Z with $\pi_1(Z) = \Gamma$ (e.g.: $Z = M$, or $Z = B\Gamma$):
  $\cdots \to \Omega^{\mathrm{spin}}_{n+1}(Z) \to R^{\mathrm{spin}}_{n+1}(Z) \to \mathrm{Pos}^{\mathrm{spin}}_n(Z) \to \Omega^{\mathrm{spin}}_n(Z) \to \cdots$
- P. and Schick proved that one can map the Stolz' surgery sequence to the Higson-Roe surgery sequence (2014)
- P. and Albin proved that one can map the Browder-Quinn surgery sequence for a Witt pseudomanifold to the Higson-Roe surgery sequence (2022)

Goals

- We want to extract numeric invariants out of these K-theory classes
- for the fundamental class $[D] \in K_*(M)$ we have the (homology) Chern character $\mathrm{Ch}_* : K_*(M) \to H_*(M, \mathbb{Q})$ and we can pair with cohomology to obtain numbers
- for the index class $\mathrm{Ind}_\Gamma(\widetilde{D}) \in K_*(C^*_r\Gamma)$ we have the seminal work of Connes and Moscovici
- we shall recall the work of Connes and Moscovici in the next slides
- the real problem is how to extract numeric invariants out of the rho class $\rho(\widetilde{D}) \in \mathrm{S}^\Gamma_*(\widetilde{M})$

Higher indices: variations on Connes-Moscovici

- we assume that M is even dimensional
- given $[\varphi] \in H^k(\Gamma)$ we have a cyclic class $[\tau^\Gamma_\varphi] \in HC^k(\mathbb{C}\Gamma)$ given by the cyclic cocycle:
  $\tau^\Gamma_\varphi(g_0, g_1, \ldots, g_k) = 0$ if $g_0 \cdots g_k \neq e$
  $\tau^\Gamma_\varphi(g_0, g_1, \ldots, g_k) = \varphi(g_0, g_0 g_1, \ldots, g_0 \cdots g_k)$ if $g_0 \cdots g_k = e$
- by definition $[\tau^\Gamma_\varphi] \in HC^*(\mathbb{C}\Gamma, \langle e \rangle)$
- in fact $H^*(\Gamma) \ni [\varphi] \to [\tau^\Gamma_\varphi] \in HC^*(\mathbb{C}\Gamma, \langle e \rangle)$ is an isomorphism
- recall that $HC^*(\mathbb{C}\Gamma)$ decomposes as the direct product of $HC^*(\mathbb{C}\Gamma, \langle x \rangle)$
- here $HC^*(\mathbb{C}\Gamma, \langle x \rangle)$ is defined requiring $\tau(g_0, g_1, \ldots, g_k) = 0$ if $g_0 \cdots g_k \notin \langle x \rangle$
- Burghelea's Theorem: $HC^*(\mathbb{C}\Gamma, \langle x \rangle)$ is explicitly computable

Variations on Connes-Moscovici (cont)

- there exists a smooth ($:=$ dense holomorphically closed) subalgebra $\mathcal{B}\Gamma$ of $C^*_r\Gamma$ defined by Connes and Moscovici
- there exists a smooth index class $\mathrm{Ind}^{\Gamma}_{\infty}(\widetilde{D}) \in K_*(\mathcal{B}\Gamma)$
- we can consider $H_*(\mathcal{B}\Gamma)$, the non-commutative de Rham homology of $\mathcal{B}\Gamma$
- there exists a Chern character $\mathrm{Ch}_{\Gamma} : K_0(\mathcal{B}\Gamma) \to H_{2*}(\mathcal{B}\Gamma)$ defined à la Chern-Weil
- there is natural pairing $\langle\, , \rangle : H_*(\mathcal{B}\Gamma) \times HC^*(\mathcal{B}\Gamma) \to \mathbb{C}$
- unfortunately we know very little about $HC^*(\mathcal{B}\Gamma)$
- instead we would like to use $HC^*(\mathbb{C}\Gamma, \langle e \rangle) = H^*(\Gamma)$
- this is indeed possible and achieved in two steps if, in addition, Γ is Gromov hyperbolic

Gromov hyperbolic groups

We fix a word length L on Γ

Step 1. Thanks to the work of Gromov one can prove $HC^*(\mathbb{C}\Gamma, \langle e \rangle)(= H^*(\Gamma))$ is equal to $HC^*_{pol}(\mathbb{C}\Gamma, \langle e \rangle)(= H^*_{pol}(\Gamma))$, the (cyclic) cocycles of polynomial growth, if Γ is Gromov hyperbolic

Step 2. Gromov Hyperbolic groups satisfy the Rapid Decay condition:
$H^\infty_L(\Gamma) := \{f \in \ell^2(\Gamma) : \sum_\gamma |f(\gamma)|^2 (1 + L(\gamma))^{2k} < +\infty \ \forall k\}$
endowed with the seminorms $\nu_k(f) := \|f(1+L)^k\|$ is continuously contained in $C^*_r\Gamma$.

Step 1+ Step 2 $\Rightarrow$ extendability of $[\tau^\Gamma_\varphi]$ from $\mathbb{C}\Gamma$ to $\mathcal{B}\Gamma$:
The RD condition implies that $H^\infty_L(\Gamma)$ is dense and holomorphically closed in $C^*_r\Gamma$ and that a cocycle of polynomial growth extends from $\mathbb{C}\Gamma$ to $H^\infty_L(\Gamma)$; but one proves that the seminorms of $H^\infty_L(\Gamma)$ are continuous on the algebra $\mathcal{B}\Gamma$ of Connes-Moscovici.

We can define the higher index $\mathrm{Ind}^\Gamma_\varphi(\widetilde{D})$, $[\varphi] \in HC^*(\mathbb{C}\Gamma, \langle e \rangle)$ and Connes-Moscovici give a formula for this higher index.

Higher indices of G-proper manifolds: geometric set-up

- G a connected semisimple Lie group (OK also real reductive Lie group)
- $K < G$ maximal compact subgroup
- (X, h), a cocompact G-proper manifold, $\dim X$ even, $\partial X = \emptyset$, with a G-invariant riemannian metric h
- proper: the map $G \times X \to X \times X$, $(g, x) \to (x, gx)$ is proper
- D, a $\mathbb{Z}_2$-graded odd G-equivariant Dirac operator acting on the sections of a G-equivariant vector bundle $E = E^+ \oplus E^-$

Higher indices on G-proper manifolds: index class

- we consider the Harish-Chandra Schwartz algebra $\mathcal{C}(G)$
- $C_c^\infty(G) \subset \mathcal{C}(G) \subset C_r^*(G)$
- $\mathcal{C}(G)$ is made of functions of "rapid decay"
- there exists a smooth index class $\mathrm{Ind}_\infty(D) \in K_*(\mathcal{C}(G)) = K_0(C_r^* G)$
- we want to extract numbers out of this index class
- there exists a Chern-Connes character with values in $HC_{\mathrm{even}}(\mathcal{C}(G))$
- there exists a pairing $HC_{\mathrm{even}}(\mathcal{C}(G)) \times HC^{\mathrm{even}}(\mathcal{C}(G)) \to \mathbb{C}$

Cyclic cocycles from group cocycles

- in the discrete case we considered a morphism $H^*(\Gamma) \to HC^*(\mathbb{C}\Gamma)$
- in the proper case there is a morphism $H^*_{\rm diff}(G) \ni [\varphi] \longrightarrow [\tau_\varphi] \in HC^*(C^\infty_c(G))$
- go back to the pairing $HC_{\rm even}(\mathcal{C}(G)) \times HC^{\rm even}(\mathcal{C}(G)) \to \mathbb{C}$
- we want classes in $HC^{\rm even}(\mathcal{C}(G))$ from our classes $[\tau_\varphi] \in HC^{\rm even}(C^\infty_c(G))$, $\varphi \in H^*_{\rm diff}(G)$
- It is again a problem of extendability, from $C^\infty_c(G)$ to the Harish-Chandra algebra $\mathcal{C}(G)$.

Extending cyclic cocycles from $C_c^\infty(G)$ to $\mathcal{C}(G)$

- We can give again the definition of Rapid Decay, with L the length function associated to a left-invariant metric on G
- (just substitute sums with integrals)
- if G satisfies the Rapid Decay condition then $H_L^\infty(G)$ is dense and holomorphically closed in $C_r^* G$. Moreover $\mathcal{C}(G)$ is a subalgebra of $H_L^\infty(G)$.
- we would like to proceed as in Connes-Moscovici and extend cyclic cocycles from $C_c^\infty(G)$ to $H_L^\infty(G)$
- we need the analogue of the result on $H^*(\Gamma)$ for Γ Gromov hyperbolic (that is, $H^*(\Gamma) = H^*_{pol}(\Gamma)$) .
- this is the content of the next result, which is a result in riemannian geometry

Extending cyclic cocycles from $C_c^\infty(G)$ to $\mathcal{C}(G)$ (cont.)

Theorem

(P-Posthuma (AKT, 2019)) if G/K has nonpositive sectional curvature then $\forall \alpha \in H^*_{\mathrm{diff}}(G)$ $\exists$ φ of polynomial growth such that $\alpha = [\varphi]$.

Example

G connected semisimple satisfies this condition and satisfies also the Rapid Decay condition

Consequently for such groups the C^*-higher index $\mathrm{Ind}_\varphi(D)$ is well-defined for each $[\varphi] \in H^*_{diff}(G)$.

There is an index formula ! (Pflaum-Posthuma-Tang)

Secondary invariants: toward a different description of Higson-Roe

- let us go back to Galois coverings
- we are heading toward a different description of the Higson-Roe surgery sequence in order to define numeric invariants out of the rho class $\rho(\widetilde{D})$
- On $\widetilde{M}$ we have
$$0 \to \Psi^{-1}_{\Gamma,c}(\widetilde{M}) \to \Psi^{0}_{\Gamma,c}(\widetilde{M}) \xrightarrow{\sigma} C^\infty(S^*M) \to 0$$
- define $\Psi^0_\Gamma(\widetilde{M}) := \overline{\Psi^0_{\Gamma,c}(\widetilde{M})}$, the C^* closure in $\mathcal{B}(L^2)$
- define $C^*(\widetilde{M})^\Gamma := \overline{\Psi^{-1}_{\Gamma,c}(\widetilde{M})}$, the C^* closure in $\mathcal{B}(L^2)$ (this is the Roe algebra)
- the C^* closure of the sequence in $\mathcal{B}(L^2)$ is:
$$0 \to C^*(\widetilde{X})^\Gamma \to \Psi^0_\Gamma(\widetilde{X}) \xrightarrow{\sigma} C(S^*X) \to 0$$

Zenobi's description of the Higson-Roe surgery sequence

Recall the short exact sequence

$$0 \to C^*(\widetilde{M}) \to \Psi^0_\Gamma(\widetilde{M}) \xrightarrow{\sigma} C(S^*M) \to 0$$

There are natural homomorphisms of algebras

- $\mathfrak{m} : C(M) \to \Psi^0_\Gamma(\widetilde{M})$ (lift and multiply)
- $\pi^* : C(M) \to C(S^*M)$ with $\pi \colon S^*M \to M$ the natural projection.

Theorem (Zenobi, 2018) *There is an isomorphism between the Higson-Roe sequence and the sequence of relative groups*

$$\cdots\cdots \xrightarrow{\delta} K_*(0 \hookrightarrow C^*(\widetilde{M}^\Gamma)) \to K_*(C(M) \xrightarrow{\mathfrak{m}} \Psi^0_\Gamma(\widetilde{M})) \xrightarrow{\sigma}$$
$$\xrightarrow{\sigma} K_*(C(M) \xrightarrow{\pi^*} C(S^*M)) \xrightarrow{\delta} \cdots$$

- in particular $\mathrm{S}^\Gamma_*(\widetilde{X}) = K_*(C(X) \xrightarrow{\mathfrak{m}} \Psi^0_\Gamma(\widetilde{X}))$

Higher rho numbers

- if $\mathcal{A}\Gamma$ is a smooth subalgebra of $C^*_r\Gamma$ we can define a smooth subalgebra subalgebra $\Psi^0_{\mathcal{A}\Gamma}(\widetilde{M})$ in $\Psi^0_\Gamma(\widetilde{M})$
- then

$$K_*(C(X) \xrightarrow{\mathfrak{m}} \Psi^0_\Gamma(\widetilde{M})) = K_*(C(X) \xrightarrow{\mathfrak{m}} \Psi^0_{\mathcal{A}\Gamma}(\widetilde{M}))$$

- Now we want to extract numbers out of the rho class $\rho(\widetilde{D})$ in $K_*(C(X) \xrightarrow{\mathfrak{m}} \Psi^0_{\mathcal{A}\Gamma}(\widetilde{M}))$
- These are higher rho numbers.
- using Lott's superconnection (a noncommutative analogue of Bismut superconnection) we define a delocalized Chern character

$$\mathrm{Ch}^{del}_\Gamma : K_*(C(X) \xrightarrow{\mathfrak{m}} \Psi^0_{\mathcal{A}\Gamma}(\widetilde{M})) \to H^{del}_{[*-1]}(\mathcal{A}\Gamma)$$

Higher rho numbers: Gromov hyperbolic groups

- Let M be odd dimensional. Assume that $\widetilde{D}$ is L^2-invertible.
- We have the rho class $\rho(\widetilde{D})$ and we've defined $\mathrm{Ch}^{del}_{\Gamma}(\rho(\widetilde{D})) \in H^{del}_{\mathrm{ev}}(\mathcal{A}\Gamma) \subset H_{\mathrm{ev}}(\mathcal{A}\Gamma)$
- We know that $H_*(\mathcal{A}\Gamma)$ embeds in $HC_*(\mathcal{A}\Gamma)$
- We would like to pair $\mathrm{Ch}^{del}_{\Gamma}(\rho(\widetilde{D})) \in H^{del}_{\mathrm{ev}}(\mathcal{A}\Gamma)$ with the delocalized cyclic cohomology groups $HC^*(\mathbb{C}\Gamma, \langle x \rangle)$, $x \neq e$.
- This is an **extension** problem as for Connes and Moscovici
- Given $[\tau] \in HC^*(\mathbb{C}\Gamma, \langle x \rangle)$ we would like to extend it to a class in $HC^*(\mathcal{A}\Gamma)$ and then use $H_*(\mathcal{A}\Gamma) \times HC^*(\mathcal{A}\Gamma) \to \mathbb{C}$
- this would give a sense to $\langle \mathrm{Ch}^{del}_{\Gamma}(\rho(\widetilde{D})), [\tau] \rangle$
- We assume Γ Gromov hyperbolic.

Higher rho numbers: Gromov hyperbolic groups (cont)

Note that this is a difficult problem already for the delocalized trace associated to $\langle x \rangle$, $\tau_{\langle x \rangle}(\sum_\gamma a_\gamma \gamma) := \sum_{g \in \langle x \rangle} a_g$

Theorem

(Puschnigg, 2010) Let Γ be Gromov hyperbolic. There exists a smooth subalgebra $\mathcal{A}\Gamma \subset C^*_r\Gamma$ s. t. $\tau_{\langle x \rangle}$ extends from $\mathbb{C}\Gamma$ to $\mathcal{A}\Gamma$.

Theorem

(P-Schick-Zenobi, 2019) Let Γ be Gromov hyperbolic. Then
(1) *$\forall x \in \Gamma$ there are isomorphisms*

$$HH^*(\mathbb{C}\Gamma, \langle x \rangle) = HH^*_{\mathrm{pol}}(\mathbb{C}\Gamma, \langle x \rangle)\,, \quad HC^*(\mathbb{C}\Gamma, \langle x \rangle) = HC^*_{\mathrm{pol}}(\mathbb{C}\Gamma, \langle x \rangle)$$

(2) *The cyclic cochains of polynomial growth extends to the Puschnigg's algebra $\mathcal{A}\Gamma$ inducing an injection $HC^*(\mathbb{C}\Gamma, \langle x \rangle) \to HC^*(\mathcal{A}\Gamma)$ as a direct summand.*

For (1) we build on results of Dan Burghelea and Ralph Meyer.
For (2) we use heavily the work of Michael Puschnigg.

Higher rho numbers: summary

- Summarizing: for a Gromov hyperbolic group we have defined the higher rho numbers
  $$\rho^\tau(\widetilde{D}) := \langle \mathrm{Ch}^{del}_\Gamma(\rho(\widetilde{D})), \tau\rangle \,, \quad \tau \in HC^*(\mathbb{C}\Gamma, \langle x\rangle).$$
- Example 1: if g is a positive scalar curvature metric and M is spin, then we have $\rho^\tau(g)$
- Example 2: if $f : X \to Y$ is a oriented homotopy equivalence and $M = X \sqcup (-Y)$ then we have $\rho^\tau(f)$ defined via $\widetilde{D}^{\mathrm{sign}}_M + A(f)$, with $A(f)$ the Hilsum-Skandalis perturbation

Simple examples

- There are explicit formulae. For example: if M is odd dimensional and τ is the delocalized trace $\tau \equiv \tau_{\langle x \rangle}$
  $\tau_{\langle x \rangle}(\sum_\gamma a_\gamma \gamma) := \sum_{g \in \langle x \rangle} a_g$
  then $\rho^\tau(\widetilde{D})$ is Lott's delocalized eta invariant $\eta_{\langle x \rangle}(\widetilde{D})$:

$$\rho^\tau(\widetilde{D}) = \eta_{\langle x \rangle}(\widetilde{D}) := \int_0^\infty t^{-1/2} \mathrm{Tr}_{\langle x \rangle}(\widetilde{D} \exp(-(t\widetilde{D})^2))dt$$

$$\mathrm{Tr}_{\langle x \rangle}(\widetilde{K}) = \sum_{\gamma \in \langle x \rangle} \int_{\mathcal{F}} \mathrm{tr}_p(\widetilde{K}(p, \gamma p)) d\mathrm{vol}\,.$$

- in particular we have
  - $\eta_{\langle x \rangle}(g)$ with M spin and g of PSC
  - $\eta_{\langle x \rangle}(f)$ if $f : X \to Y$ is an oriented homotopy equivalence

Bordism invariance

- If M is closed then $\mathrm{Ind}^\tau(\widetilde{D}) = 0 \ \forall \tau \in HC^*(\mathbb{C}\Gamma, \langle x \rangle)$, $x \neq 0$
- if our manifold, W, has a boundary this is not so
- there exists a delocalized higher APS index theorem on a Galois covering with boundary $\Gamma \to \widetilde{W} \to W$, assuming the boundary operator invertible (P-Schick):
  $$\mathrm{Ind}^\tau_{\mathrm{APS}}(\widetilde{D}_W) = -\frac{1}{2}\rho^\tau(\widetilde{D}_{\partial W})$$
- this implies that $\rho^\tau(g)$ descends to $\mathcal{P}^+(M)$, the set of concordance classes of PSC-metrics
- similarly $\rho^\tau(f_1) = \rho^\tau(f_2)$ if $f_1 : N_1 \to M$ and $f_2 : N_2 \to M$ are h-cobordant
- many interesting geometric applications

More higher rho numbers

- In addition to

$$\langle\ ,\ \rangle : \mathrm{S}^{\Gamma}_{*}(\widetilde{M}) \times HC^{*}(\mathbb{C}\Gamma, \langle x \rangle) \to \mathbb{C}$$

  we also define in our paper a pairing

$$\langle\ ,\ \rangle : \mathrm{S}^{\Gamma}_{*}(\widetilde{M}) \times H^{*}(M \to B\Gamma) \to \mathbb{C}$$

  if Γ satisfies the Rapid Decay condition and $H^{*}(\Gamma) = H^{*}_{pol}(\Gamma)$ (for example Gromov hyperbolic groups).
- all of these results in the article *Mapping analytic surgery to homology, higher rho numbers and metrics of positive scalar curvature* (P -Schick- Zenobi), to appear in Memoirs AMS
- related results also by Chen-Wang-Xie-Yu and Weinberger-Xie-Yu

A glimpse to the G-proper case

- Given $g \in G$ a semisimple element we consider the so called orbital integral tr_g: if $Z := Z_G(g)$ and $f \in C_c^\infty(G)$ then

$$\mathrm{tr}_g(f) := \int_{G/Z} f(xgx^{-1}) d(xZ)\,.$$

- Proposition: tr_g is a trace and it extends to $\mathcal{C}(G)$
- from tr_g we define a trace tr_g^X on $\Psi^{-\infty}_{G,c}(X)$:

$$\mathrm{tr}_g^X(k) := \int_{G/Z}\int_X c(hgh^{-1}x)\mathrm{tr}(hgh^{-1}\kappa(hg^{-1}h^{-1}x,x))dx\, d(hZ)$$

- it extends to "Harish-Chandra smoothing operators"

Questions

- can we define a delocalized eta invariant $\eta_g(D)$ using the heat kernel and tr_g^X ?
- if C is a smoothing perturbation can we define $\eta_g(D+C)$?
- in particular, can we define the delocalized eta invariant of a PSC metric g and of a G-equivariant homot. equivalence f ?
- is there a delocalized APS index theorem ?
- are there higher versions of these results ?
- together with Hessel Posthuma, Yanli Song and Xiang Tang we give positive answers to these questions
- contributions also by Peter Hochs, Bai-Ling Wang and Hang Wang
- our work in the preprint *Higher orbital integrals, rho numbers and index theory*
- and in the new preprint *Perturbed Dirac operators and index theory on G-proper manifolds* (to appear in arxiv)
- all this in the talk of Hessel Posthuma on Friday

THANK YOU!