Pulsars with PINNs: part 1 (2D)

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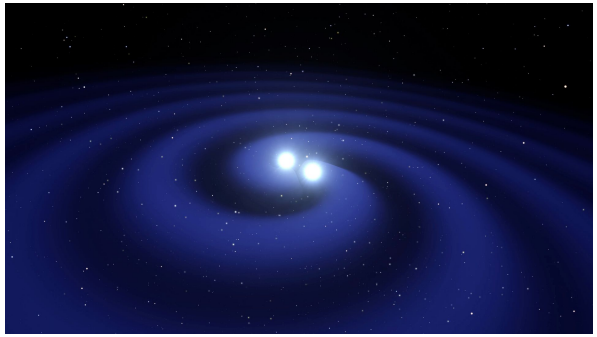
OSCARS
Open Science Clusters' Action for Research & Society

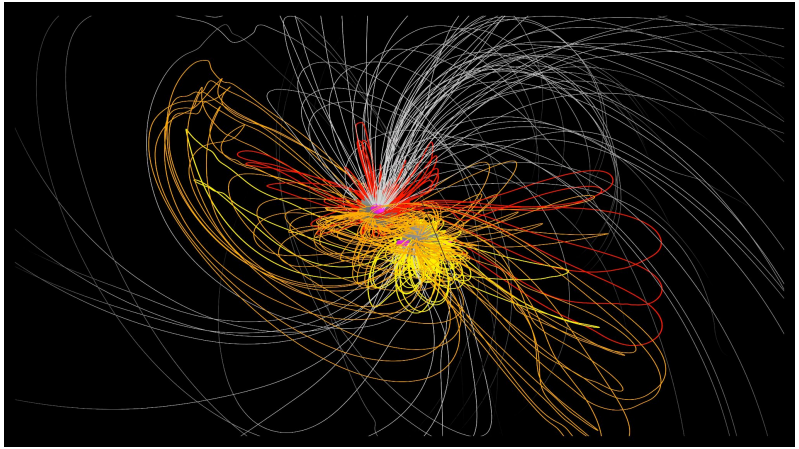
Talk outline

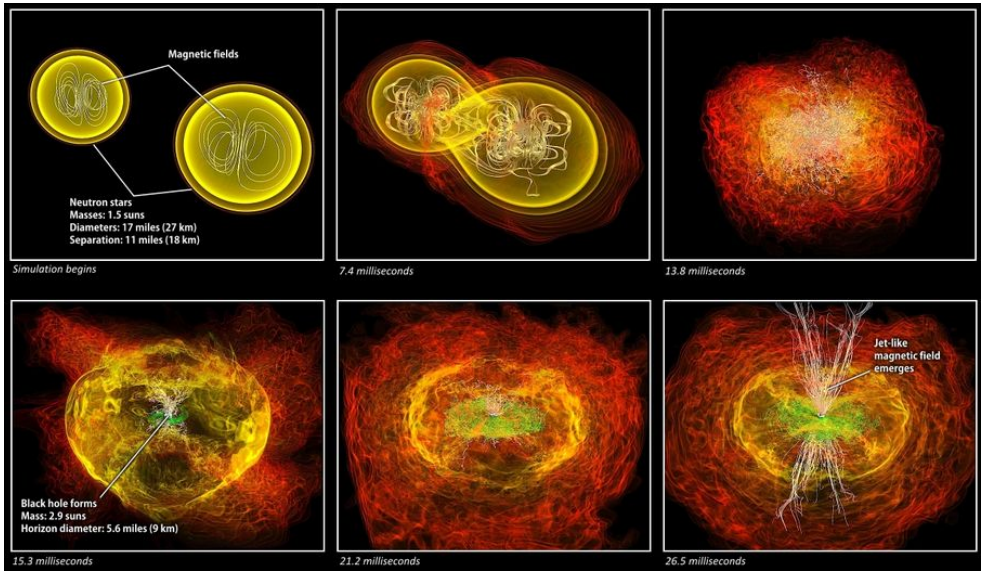
- Introduction, Pulsar in the era of Multi-Messenger astrophysics
- Pulsar magnetosphere problem and the main issues
- A novel approach
- Physics Informed Neural Networks (PINNs) as solver of Differential equations
- Results
- Summary

Introduction, Pulsar in the era of Multi-Messenger astrophysics

- Neutron star merger, gravitational waves
  e.g. GW170817, GW190425



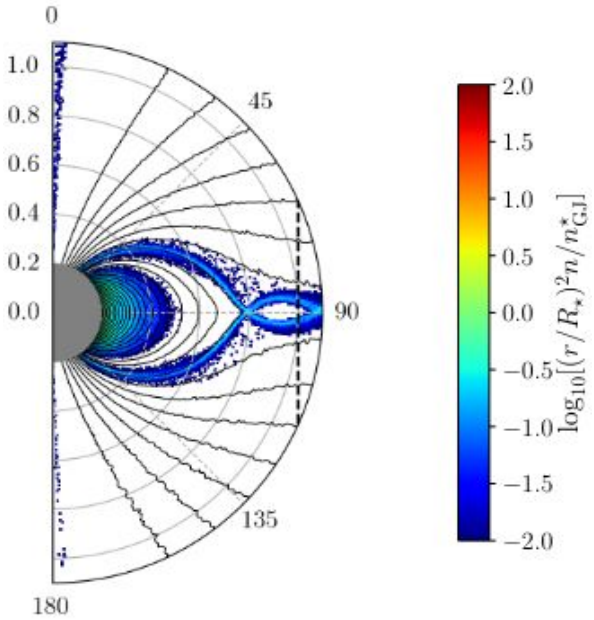


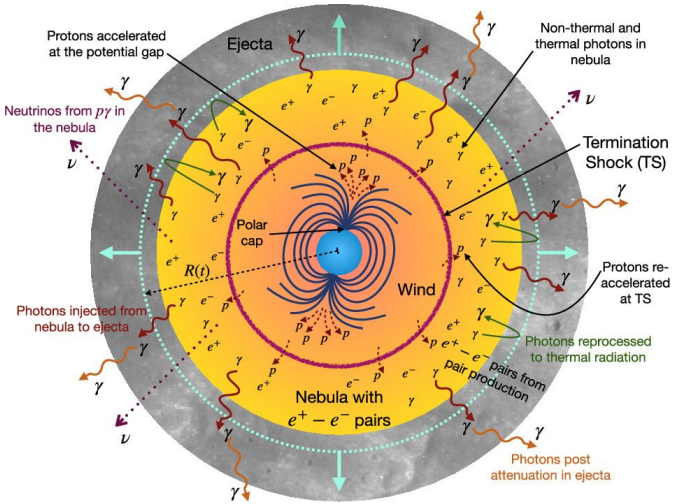


ESO/L. Calçada, Skiathos et al. 2025, NASA/AEI/ZIB/M. Koppitz and L. Rezzolla

Introduction, Pulsar in the era of Multi-Messenger astrophysics

- High-energy neutrinos
- Pulsars as potential cosmic-ray accelerators



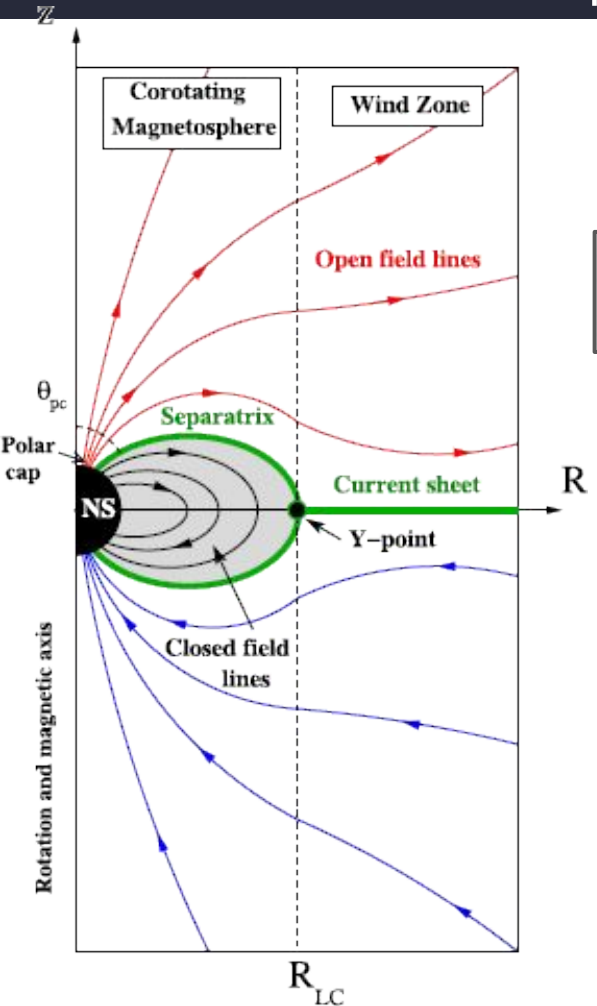


Cosmic rays and pulsars: connections from low to ultrahigh energies

Kumiko Kotera

Mukhopadhyay et al. 2025, Guepin et al. 2020, Kotcra 2019

Back to the basics- The pulsar Magnetosphere



The pulsar magnetosphere problem in 2D the Force-Free case

$$\left(1-\frac{r^2\sin^2\theta}{R_{\rm LC}^2}\right)\left[\frac{\partial^2\Psi}{\partial r^2}-\frac{\partial\Psi}{\partial\theta}\frac{\cos\theta}{r^2\sin\theta}+\frac{1}{r^2}\frac{\partial^2\Psi}{\partial\theta^2}\right]-\frac{2r\sin\theta}{R_{\rm LC}^2}\left[\frac{\partial\Psi}{\partial\theta}\frac{\cos\theta}{r}+\frac{\partial\Psi}{\partial r}\sin\theta\right]+II'(\Psi)=0$$

$$I = I(\Psi)$$

$$\frac{1}{c}\boldsymbol{J}\times\boldsymbol{B}+\rho_e\boldsymbol{E}=0$$

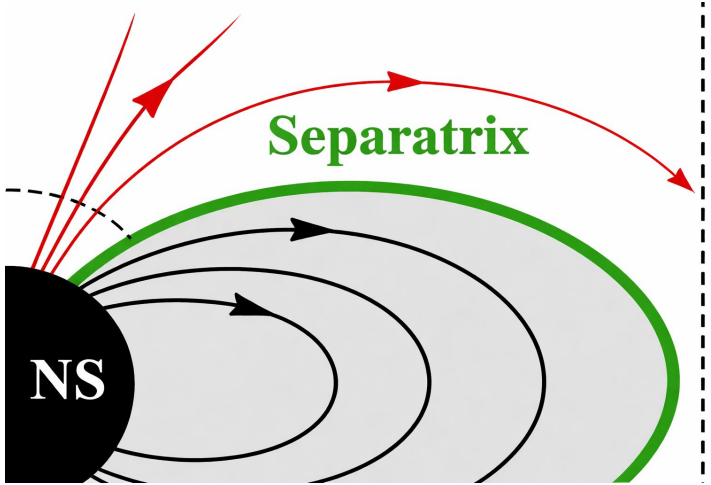
Boundary Conditions in the star: Dipole field

Contopoulos et al. 1999, Cerutti et al. 2017

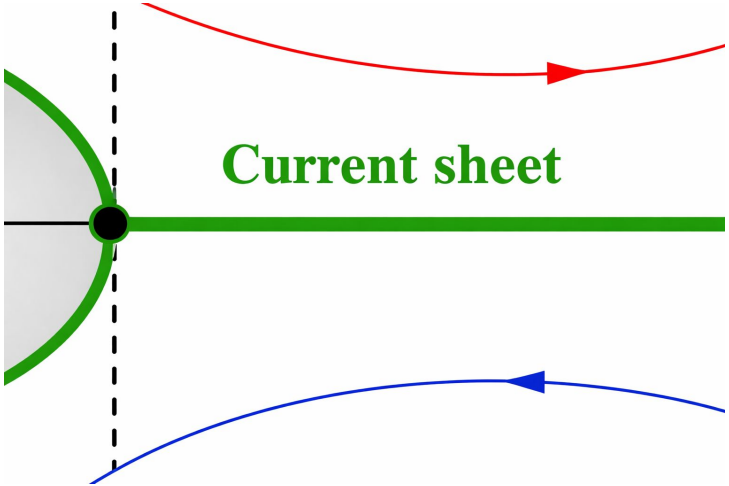
Back to the basics- The pulsar Magnetosphere

Three major issues of this configuration

1)Discontinuity of the current *I*



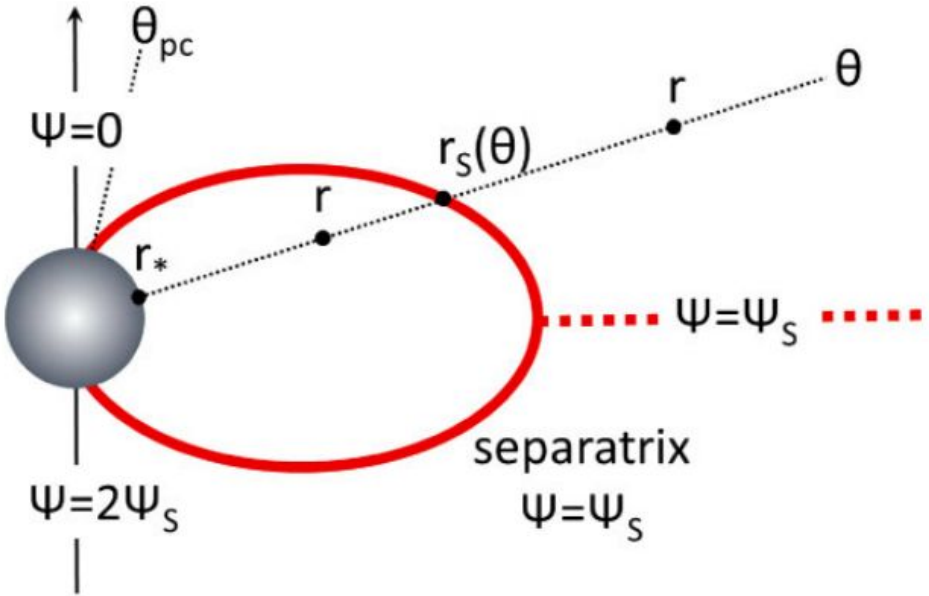
2)Discontinuity of the magnetic field at the equator



3) Current sheets are not Force-Free regions

A novel approach

We want to solve the pulsar equation with zero energy dissipation



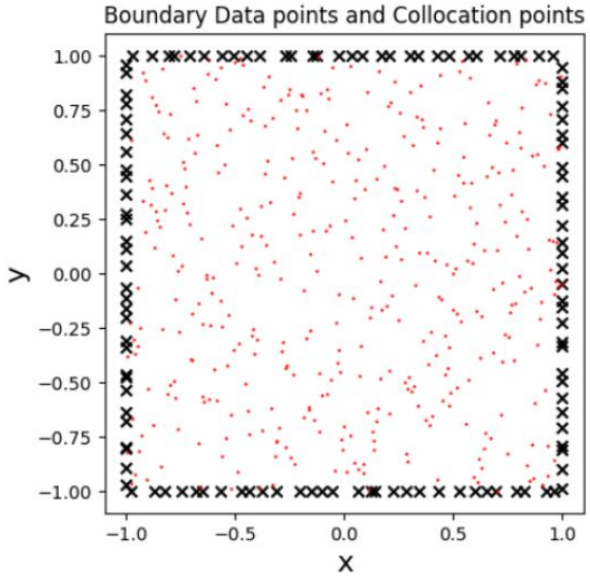
- Split the domain into two distinct regions — one associated with open magnetic field lines and the other with closed magnetic field lines. (option θ_{pc})
- Separatrix balance condition:

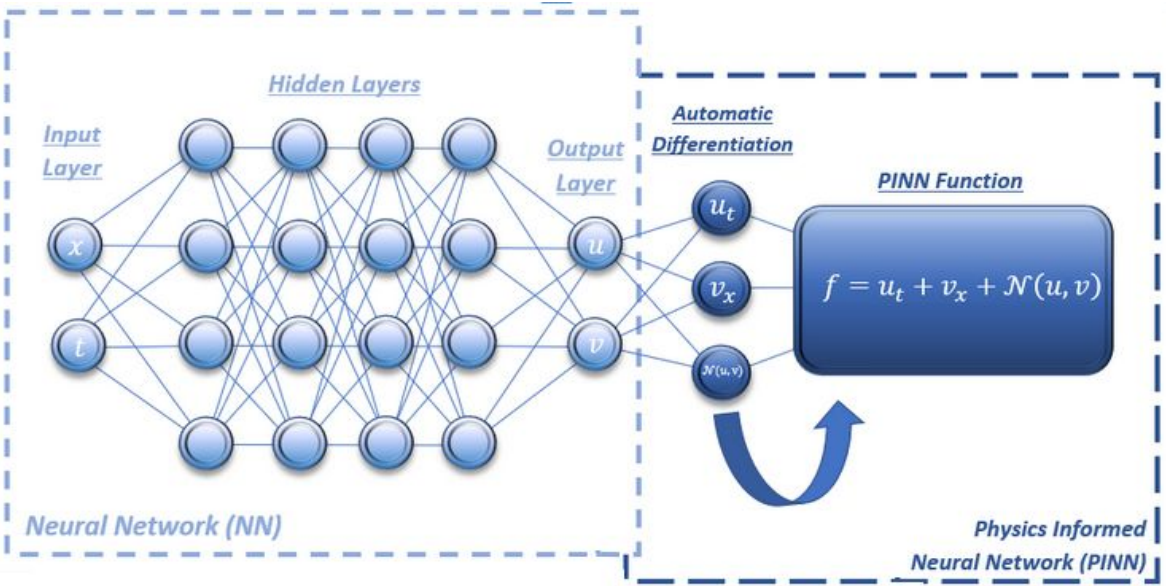
$$(B^2 - E^2)_{open} = (B^2 - E^2)_{closed}$$

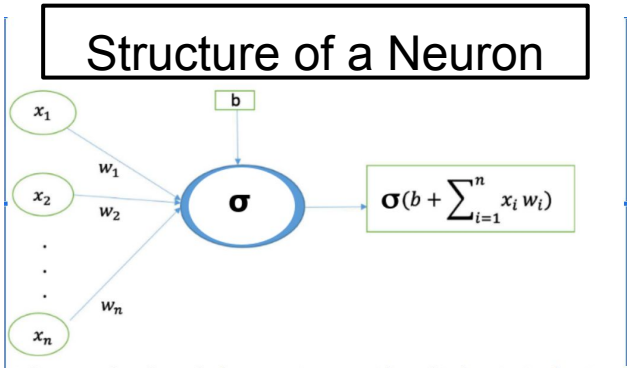
- We assume positive magnetic flux in the southern hemisphere as well.
- Solver equations: Physics Informed Neural Networks

Dimitropoulos et al. 2024

Physics Informed Neural Networks (PINNs) as solver of Differential equations







σ: Activation function (eg Relu, tanh,...)

Hubert Baty, A hands-on introduction to PINNs, 2024

Physics Informed Neural Networks (PINNs) as solver of Differential equations

$$L_{data}(\theta) = \frac{1}{N_{data}} \sum_{i=1}^{N_{data}} \left| (u_\theta(\boldsymbol{x_i}) - u_i^{data} \right|^2 .$$

$$L_{PDE}(\theta) = \frac{1}{N_c} \sum_{i=1}^{N_c} |\mathcal{F}(u_\theta(\boldsymbol{x_i}))|^2 ,$$

Example for the heat equation

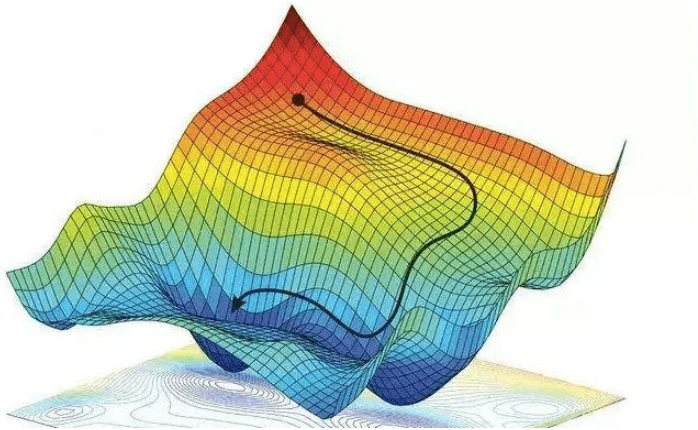
$$\frac{\partial u}{\partial t} = a\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

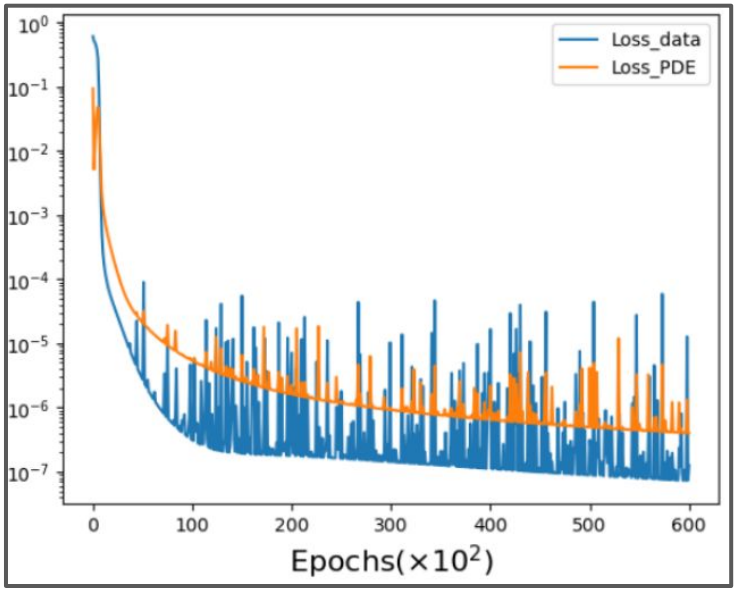
The NN gives us the function u_θ and we need:

$$\frac{\partial u_\theta}{\partial t} - a\left(\frac{\partial^2 u_\theta}{\partial x^2} + \frac{\partial^2 u_\theta}{\partial y^2} + \frac{\partial^2 u_\theta}{\partial z^2}\right) \to 0$$

so $$\mathcal{F}(u_\theta) = \frac{\partial u_\theta}{\partial t} - a\left(\frac{\partial^2 u_\theta}{\partial x^2} + \frac{\partial^2 u_\theta}{\partial y^2} + \frac{\partial^2 u_\theta}{\partial z^2}\right)$$

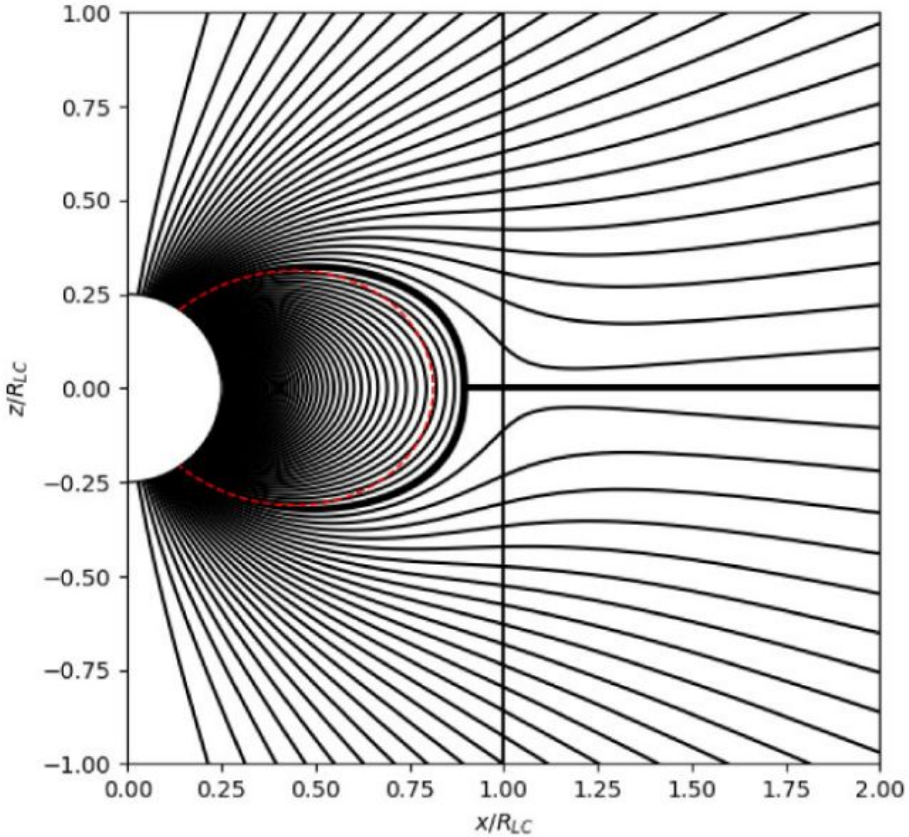
$$L(\theta) = \omega_{data} L_{data}(\theta) + \omega_{PDE} L_{PDE}(\theta),$$

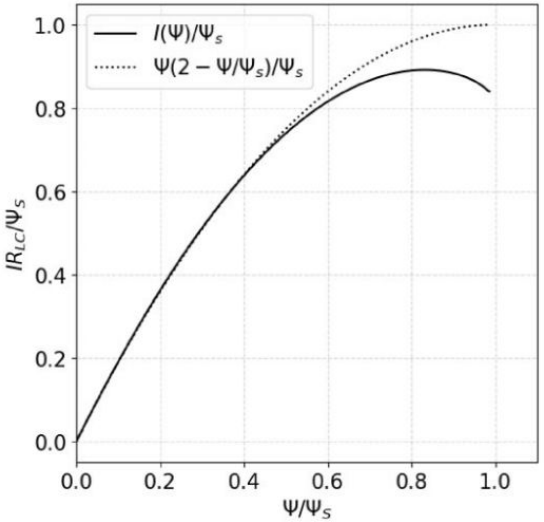




Hubert Baty, A hands-on introduction to PINNs, 2024

Results-The solution of the pulsar equation

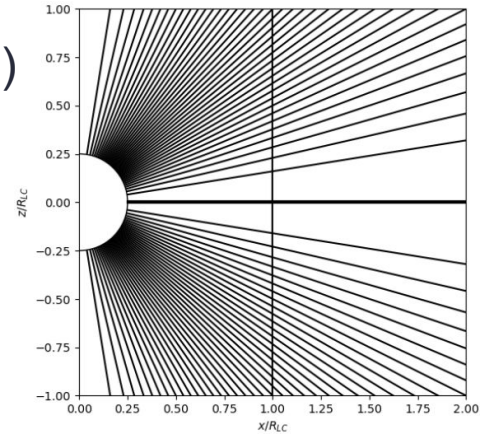




Dimitropoulos et al. 2024

Results- A family of news solutions

1)



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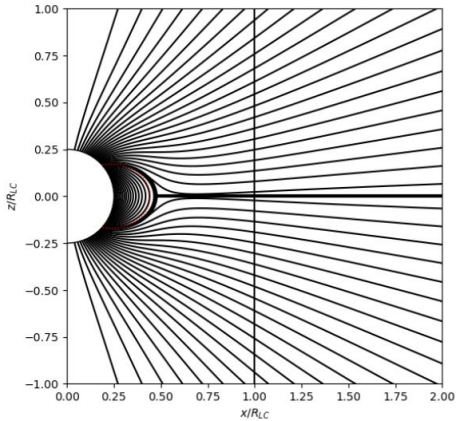

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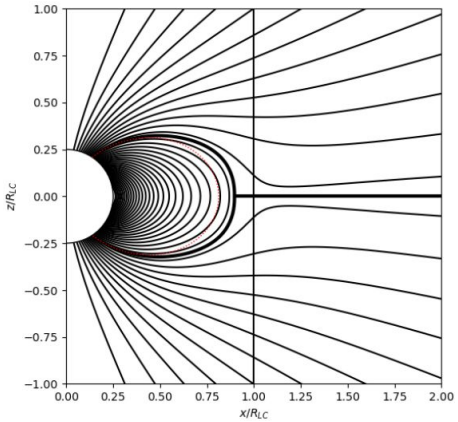
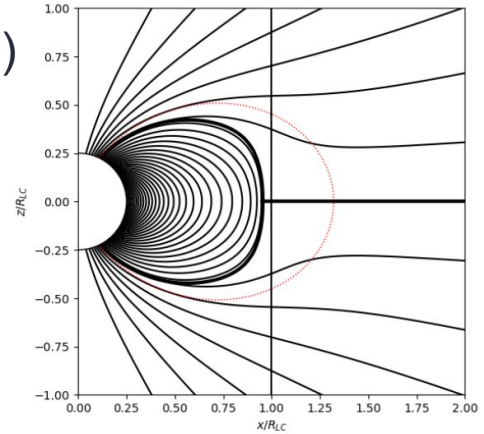


4)



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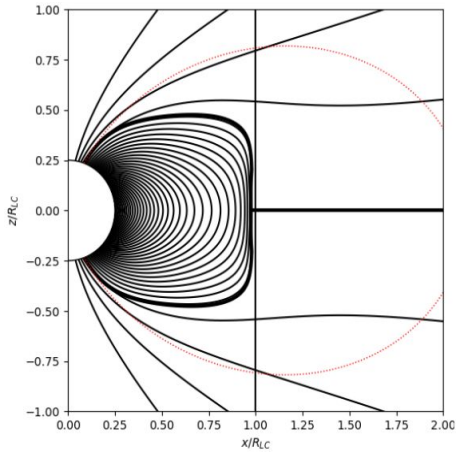

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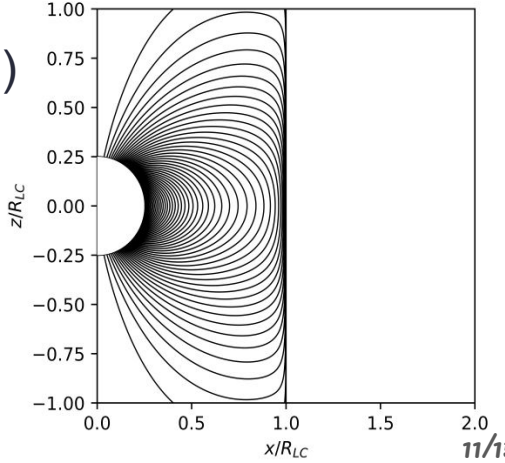
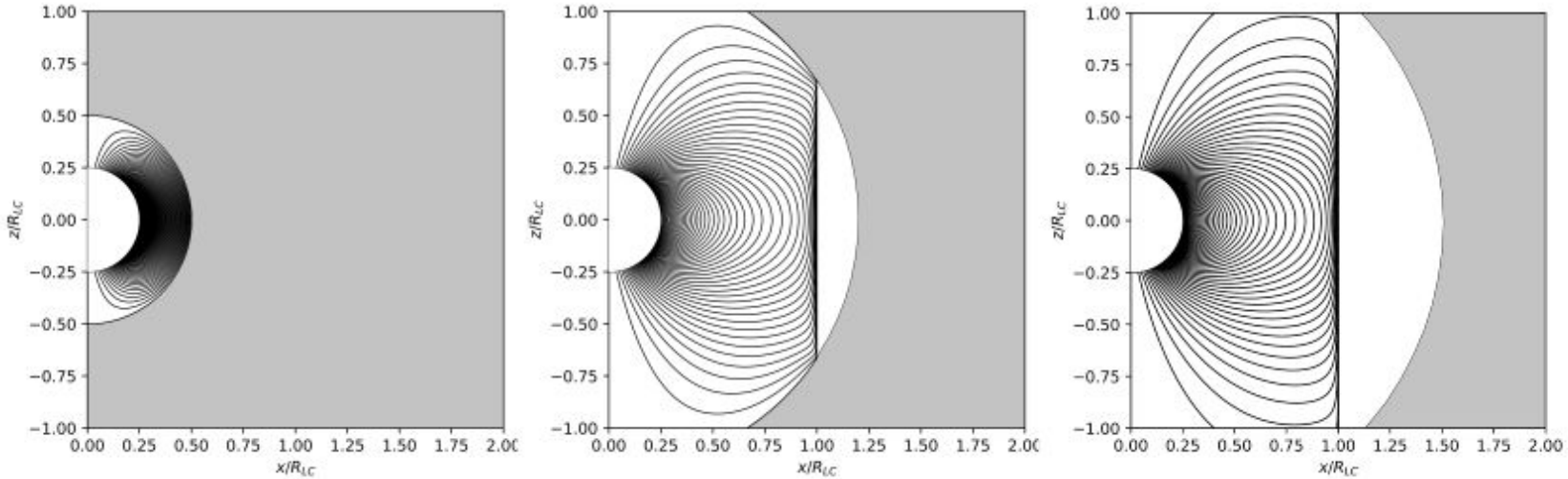
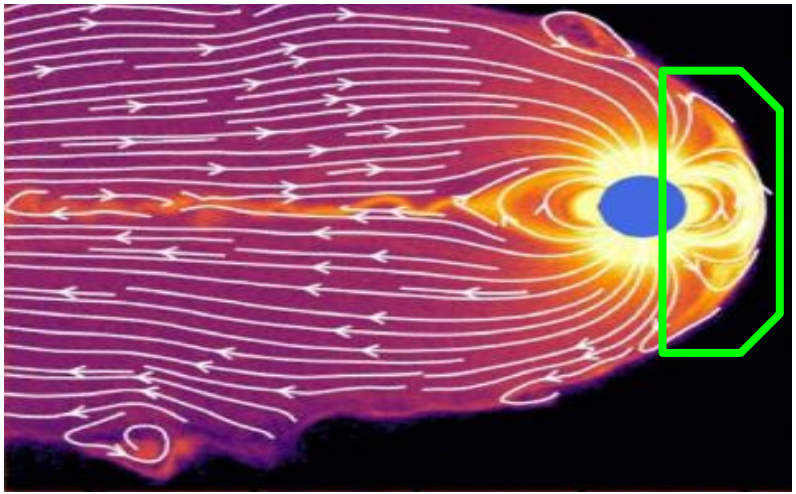


Contopoulos, Dimitropoulos et al. 2024





PIC simulation

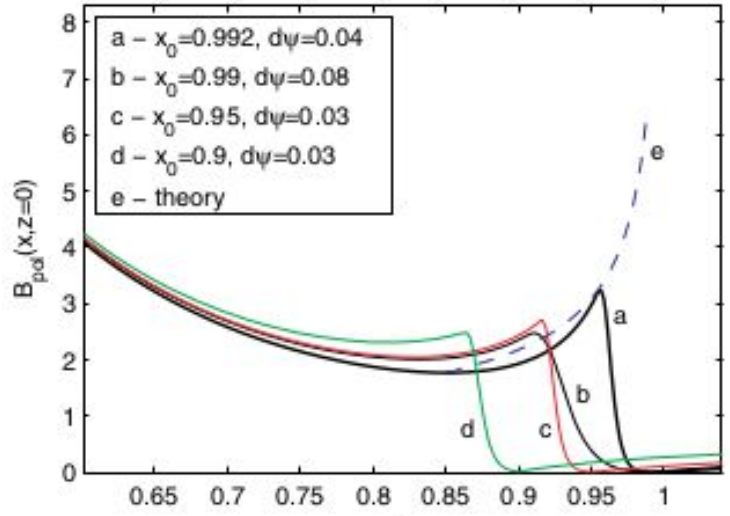
Contopoulos, Dimitropoulos et al. 2024, Yici Zhong et al. 2024

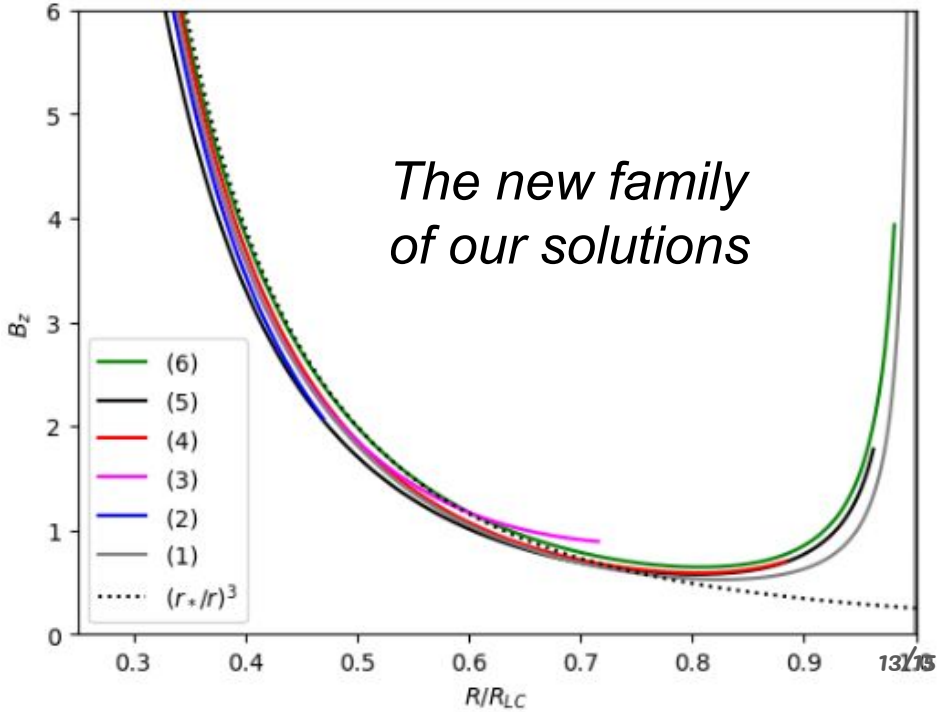
Results- A family of news solutions

$$(B^2 - E^2)_{closed} = (B^2 - E^2)_{open}$$

$$B_p(x_Y)\big|_{\text{closed}} = \frac{I(\Psi_s)}{x_Y\sqrt{1-x_Y^2}} = \frac{I(\Psi_s)}{\sqrt{2}\sqrt{1-x_Y}} \to \infty \quad \text{when } x_Y \to 1$$

High-resolution calculation, with the classic method





Contopoulos, Dimitropoulos et al. 2024, Timokhin et al. 2006

Summary

- Pulsar a promising lab for multimessenger astrophysics
- Maybe we have the basic picture of pulsars magnetosphere but we need to focus and to worry about crucial details
- The family of solutions that presented is a further step and open questions in this field of research
- PINNs a powerful solving tool but difficult to handle
- Please watch the talk of Dr. Contopoulos for part2…

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