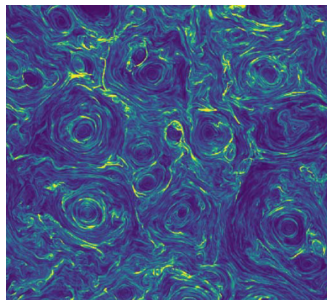


STRIPE: Modeling Relativistic Turbulent Acceleration in Extreme Astrophysical Sources

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NATIONAL ASTROPHYSICS AND
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MuMeNTA workshop (Harokopio University, Athens). 2-4 March 2026

Work based on a Master Project (NWU)

This research work was conducted within the frame of a Master Project at North-West University (NWU) involving **Frans van der Merwe**

The Master Project is supported with the funding from the National Astrophysics and Space Science Programme (NASSP) of South Africa



Stochastic (second-order Fermi) acceleration: quasi-linear framework

- Resonant interactions with *magnetic irregularities* due to **plasma waves** (e.g. Alfvén waves)
- Standard *quasi-linear framework* (QLT) (e.g. Schlickeiser (1989)) is reasonably accurate in weak turbulence regimes ($\delta B/B \ll 1$)
- Evolution of particle spectrum: **Fokker–Planck equation**

$$\frac{\partial N_e(\gamma, t)}{\partial t} = -\frac{\partial}{\partial \gamma} \left[\frac{2}{\gamma} D_{pp}(\gamma, t) N_e(\gamma, t) \right] + \frac{\partial}{\partial \gamma} \left(D_{pp}(\gamma, t) \frac{\partial N_e(\gamma, t)}{\partial \gamma} \right) - \frac{N_e(\gamma, t)}{t_{\text{esc}}(\gamma, t)} + Q_{\text{inj}}(\gamma, t)$$

- Turbulence *power spectrum* $W(k) \propto k^{-q}$
Diffusion coefficient $D_{pp}(p) \propto p^q$

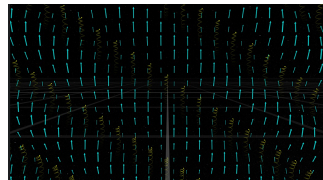
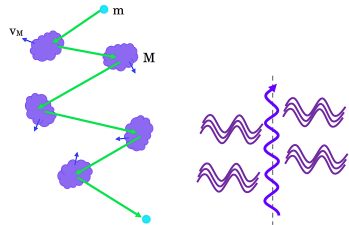


Figure: Top: (left) second-order Fermi sketch (A. Dmytriiev), (right) wave-particle interactions sketch (M. Lemoine). Bottom: illustration of Alfvén waves in plasma (credit: NASA)

High-Amplitude and Relativistic MHD turbulence

- QLT fails in the regime of **strong turbulence** ($\delta B/B \sim 1$) $\rightarrow$ *non-linear effects emerge*
- Relativistic Alfvén velocity $v_A = \sqrt{\frac{\sigma}{1+\sigma}} \sim c$
- **Nonresonant phenomena**: interaction of particles with nontrivial velocity structures
- No comprehensive physical theory of turbulence !
- Approach? $\rightarrow$ **Numerical MHD** and **Particle-in-Cell (PIC) simulations**: self-consistently follow plasma waves, velocity structures and particle dynamics in 2D/3D

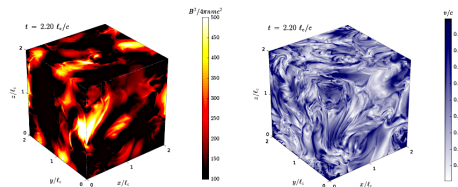
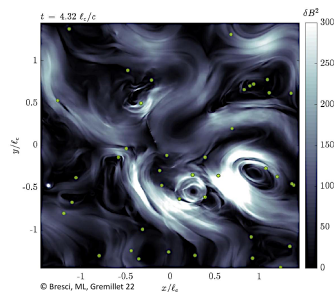


Figure: Top: turbulence simulation (Bresci; Lemoine; Gremillet 2022). Bottom: example of a PIC simulation (magnetic energy density and mean plasma velocity) (Bresci et al. (2022))

High-Amplitude Turbulence: key findings from MHD/PIC simulations

- Particle energization is dominated by *complex plasma structures* (e.g. Pugliese et al. (2023))
 - **Sharp bends of magnetic field lines**
 - **compression modes** $\perp$ to the field(Bresci et al. (2022); Lemoine et al. (2021))
- Velocity and magnetic density fluctuations become highly non-Gaussian, with **spatial intermittency** (e.g. Beattie et al. (2020))
- Particle acceleration is more intense in specific regions of space (e.g. Pezzi et al. (2022))
- Particle energization becomes **highly intermittent**, especially at small spatial scales (e.g. Nättilä & Beloborodov (2021))

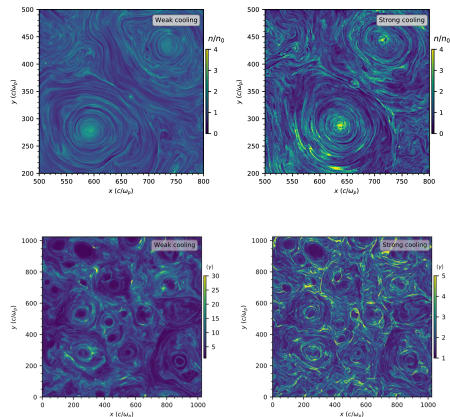
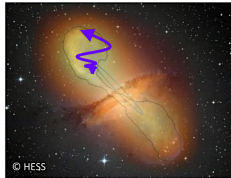
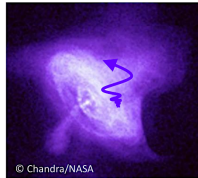


Figure: Top: PIC simulation (zoom-in) showing relative plasma density. Bottom: mean Lorentz factor in the PIC simulation – visualizes intermittency of particle energization (Nättilä & Beloborodov (2021))

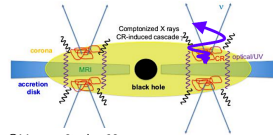
Astrophysical relevance of high-amplitude turbulent acceleration



... in extra-galactic jets



... in pulsar wind nebulae



... in black hole environments



Figure: Sources of strong MHD turbulence (credit: M. Lemoine; ESA/Hubble)

- **Large-amplitude turbulence** ($\delta B/B \sim 1$) arises when kinetic/magnetic energy is injected and *rapidly dissipated*
- **Relativistic turbulence** ($v_A \sim c$) emerges when magnetization is high, $\sigma \geq 1$
- **Pulsar Wind Nebulae**: relativistic turbulence downstream of the wind termination shock
- **Plasma jets (AGN / microquasars)**: internal shocks, magnetic reconnection, instabilities, velocity shear
- **Jet termination region**: low B in the ISM, high jet power dissipated rapidly $\rightarrow \delta B/B \sim 1$
E.g. microquasar TeV-PeV “bubbles”
- **BH coronae**: very high magnetization and efficient proton acceleration

Why modeling the acceleration process accurately is so important?

- Most modeling efforts assume *simplistic power-law* or *log-parabola* particle spectra
- OK for *shock acceleration* in certain physical regimes
- **Very questionable for Fermi-II acceleration, especially in high-amplitude regime !**
- Detailed shape of the particle spectrum has a **MASSIVE** influence on the resulting SED
- More advanced acceleration models are needed to explain peculiar SED features: *very hard spectra, sharp breaks, narrow spectral peaks, maximum particle energies, etc*

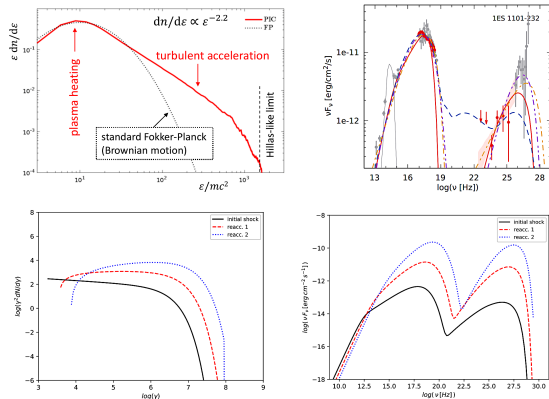


Figure: Top-left: PIC-simulated particle spectrum resulting from turbulent acceleration (credit: M. Lemoine). Top-right: example of a very hard extreme blazar SED (Goswami et al. (2024)). Bottom: particle spectra and corresponding SEDs resulting from acceleration on successive shocks (Zech & Lemoine (2021))

Constructing a generalized framework

MHD/PIC simulations:

- **Advantage:** from first principles
- **Drawback:** computationally costly !
- **Approach?** → develop physically-motivated self-consistent *theoretical frameworks* that **condense** key patterns observed in MHD/PIC simulations
- Predictive power: efficiently simulate particle spectra arising from turbulent acceleration for a large variety of parameters

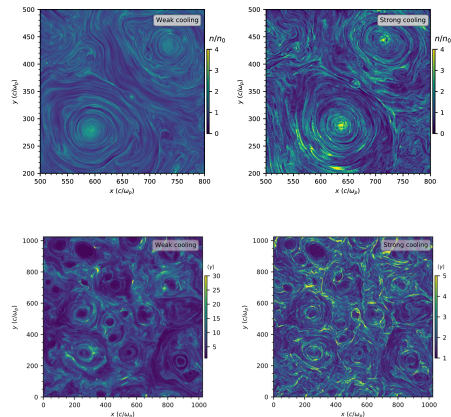


Figure: Top: PIC simulation (zoom-in) showing relative plasma density. Bottom: mean Lorentz factor in the PIC simulation – visualizes intermittency of particle energization (Nättilä & Beloborodov (2021))

Stochastic acceleration framework by Lemoine 2022

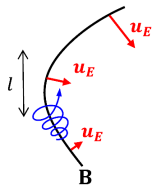
- Valid for high-amplitude regime $\delta B/B \sim 1$
- Connects the **rate of energization** to the **gradients of the velocity of magnetic field lines**
- Particles scatter in *sharp bends of magnetic field lines* and *compression modes* $\perp$ to the field
- In scattering center frame **particle momentum evolves as:**

$$\dot{p} = p \{ \Gamma_{\text{acc}} + \Gamma_{\parallel} + \Gamma_{\perp} \} = p \Gamma_l$$

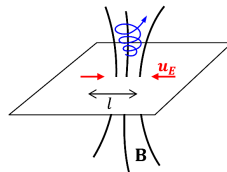
where $\Gamma_{\text{acc}/\parallel/\perp}$ is an effective inertial force due to *acceleration of magnetic field lines*, *shear along the field*, and *transverse compression*

Γ_l is the **aggregate (random) gradient**, coarse-grained on scale $l_g = 2\pi r_g$

- Gradients Γ_{l_g} are subject to *intermittency* on small length scales
 - random forces are neither Gaussian nor white noise in time
 - **random walk deviates from Brownian motion**



shear of u_E along B field
(aka curvature drift)



compression of $u_E \perp B$ field
(aka betatron, grad-B)

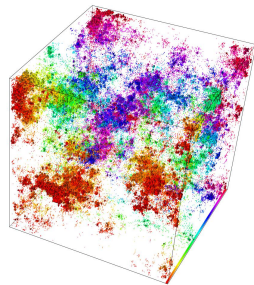
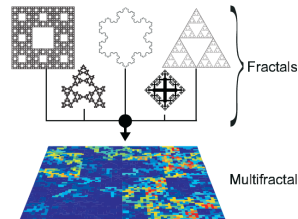
- Discrete stochastic interactions in both momentum and time
- Δp and Δt are considered **random variables**
- Δt is distributed **exponentially** around $\langle \Delta t(p) \rangle = l_g(p)/c = 2\pi r_g(p)/c$
- $\Delta \ln p$ is distributed as $\Gamma_{l_g} \Delta t$
- The PDF of Γ_{l_g} (statistics of momentum jumps): described within **multifractal analysis**:

$\Gamma_l(x) \sim \Gamma_{l_c}(x) (l/l_c)^{h(x)} \rightarrow$ describes set of locations x with index $h(x)$ as a **fractal** of dimension $d(h)$

$$p_{\Gamma_l} \sim \int d\Gamma_{l_c} p_{\Gamma_{l_c}} \int dh l^{3-d(h)} \delta \left[\Gamma_l - \Gamma_{l_c} (l/l_c)^h \right]$$

where spectrum $d(h)$ depends on physical properties of turbulence

- **Acceleration rate varies through the volume** (distributed as PL)
- **NB:** transport equation differs significantly from Fokker-Planck



Statistics of velocity gradients (Lemoine 2022)

- **Statistics of velocity gradients** Γ_l is *measured* from **numerical simulation** of driven incompressible MHD turbulence (1024^3 with 1024 time steps) (Eyink et al. (2013)) (JHU Turbulence database)
- **Approximation:** a broken power-law function to describe the PDF of Γ_l (blue curve):

$$p_{\Gamma_l} \approx \left[1 + \left(\frac{\Gamma_l}{\sigma(l)} \right)^{k_0(l)/k_1} \right]^{-k_1}$$

where $\sigma(l)$ is the width of the distribution

- **Particles can experience very large jumps in momentum/energy !** $\rightarrow$ very different from Fokker-Planck

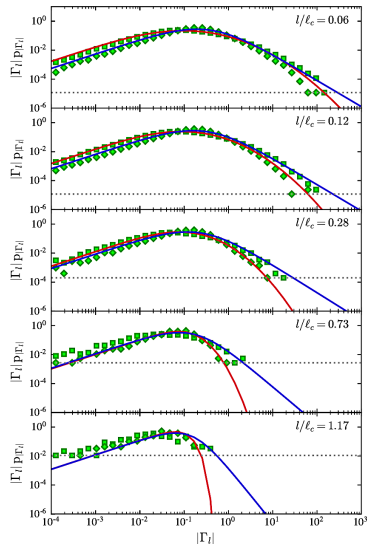


Figure: Distribution of gradients Γ_l (Lemoine 2022)

The STRIPE code: Strong-Turbulence Relativistic Intermittent Particle Energization



F. van der Merwe & A. Dmytriiev (2025)

The STRIPE code: Monte-Carlo implementation of Lemoine (2022) framework

- C/C++, parallelized computation (MPI)
- **Fundamental input parameters:**
 - Magnetic field B
 - Turbulence coherence length l_c
 - Turbulent Alfvén velocity $\beta_a = \beta_A \frac{\delta B}{B}$
 - Escape time-scale t_{esc}
 - Compton dominance CD
- Particles are **injected** *instantaneously/continuously* with certain γ_0
- **Waiting (interaction) time Δt** is drawn at random from the exponential distribution with $\langle \Delta t(p, B) \rangle = 2\pi r_g(p, B)/c$
- **Gradient** of the magnetic-field-line velocity Γ_l is drawn at random from a broken power-law approximation (with cutoff)
- **Sign of Γ_l** is drawn at random (energy gain/loss)
- Momentum p is changed at each step

Synchrotron/IC losses incorporated self-consistently



```
501 using namespace std;
502
503 void merge(int a[], int temp[], int left, int mid, int right)
504 {
505     int i = left, j = mid, k = left;
506     while (i <= mid - 1 && j <= right) {
507         if (a[i] <= a[j]) {
508             temp[k++] = a[i++];
509         } else {
510             temp[k++] = a[j++];
511         }
512     }
513     while (i <= mid - 1) temp[k++] = a[i++];
514     while (j <= right) temp[k++] = a[j++];
515     for (i = left; i <= right; i++) a[i] = temp[i];
516 }
517
518 void mergesort(int a[], int temp[], int left, int right)
519 {
520     if (left < right) {
521         int mid = left + (right - left) / 2;
522         mergesort(a, temp, left, mid);
523         mergesort(a, temp, mid + 1, right);
524         merge(a, temp, left, mid, right);
525     }
526 }
```

Examples of STRIPE-simulated electron spectra

We simulate electron spectra for typical physical parameters in microquasar “bubbles”

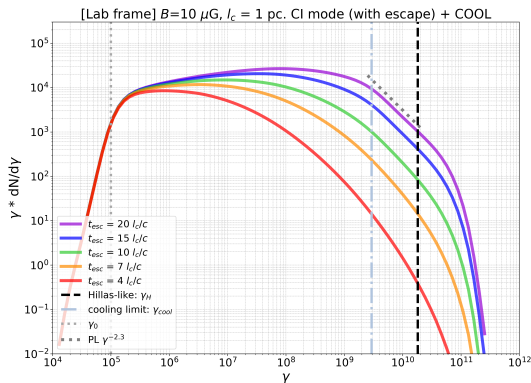
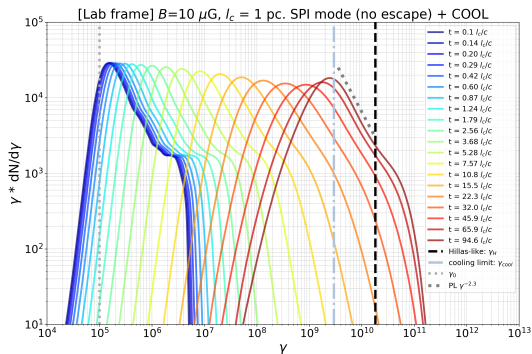
$$B = 10 \mu\text{G}, l_c = 1 \text{ pc}, \beta_a = 0.41. \gamma_0 = 10^5$$

WITH radiative cooling (synchrotron)

Instant. injection

PL tail index ≈ 3.3

Continuous injection + escape



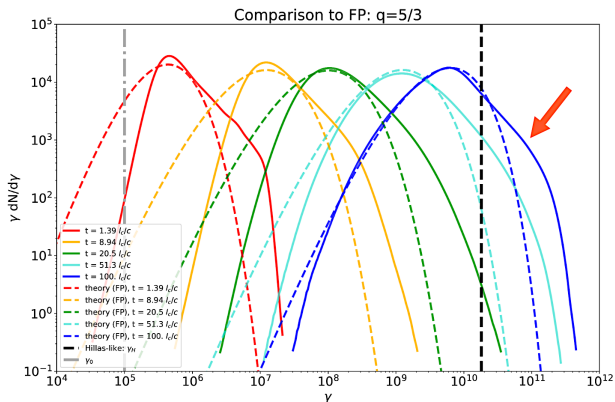
Comparison of STRIPE-simulated spectra to Fokker-Planck

We juxtapose spectra simulated with STRIPE to solutions of Fokker-Planck equation (no cooling).

Lemoine (2022) model yields **much narrower spectra** with **extended PL tails** – effect of intermittency

$$B = 10 \mu\text{G}, l_c = 1 \text{ pc}, \beta_a = 0.41. \gamma_0 = 10^5$$

NO COOLING. Pure acceleration. $q = 5/3$



Asymptotic electron spectra: dependence on Alfvén velocity β_a

$$B = 10 \mu\text{G}, l_c = 1 \text{ pc. } \gamma_0 = 10^5$$

with radiative cooling (synchrotron)

$$\beta_a = [0.3, 0.41, 0.5]$$

$$\text{Turbulent Alfvén velocity } \beta_a = \beta_A \frac{\delta B}{B}. \quad \beta_A = \sqrt{\sigma/(1+\sigma)}. \quad B = \sqrt{B_0^2 + \delta B^2}$$

Instantaneous injection

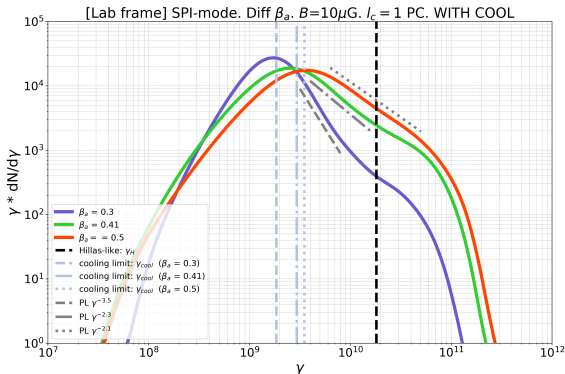
$$\text{MAX PL index} \approx 3$$

$$(\beta_a = 0.5)$$

STRONG dependence on β_a !

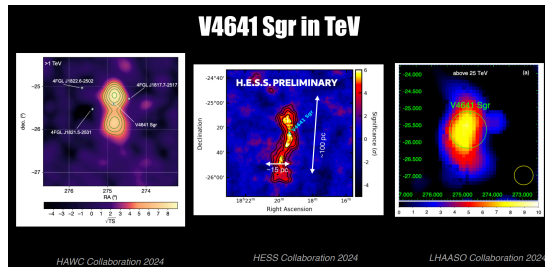
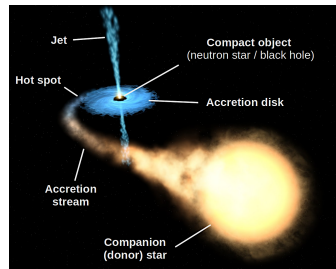
Higher β_a yields harder spectra

Weak dependence on B and l_c



Application to Astrophysical Sources: LHAASO microquasar V4641 Sgr

- Galactic microquasar V4641 Sgr was recently reported by LHAASO as a UHE ($E_\gamma > 100$ TeV) emitter, with emission up to **800 TeV**
- Hardest** LHAASO spectral index amongst microquasars: $\alpha \approx 2.67 \pm 0.27$
- Detected also by HAWC and H.E.S.S.
No detection by *Fermi*-LAT
- Extended emission on **100 pc** scale !
- Also detected in the X-ray band by XRISM (Suzuki et al. (2025))



Model for particle acceleration and MWL emission from V4641 Sgr

- Misaligned jet **terminates** at binary scales: injection of turbulence across ~ 100 pc volume (**microquasar “bubbles”**)
- **Electron spectrum:** e^- are *continuously* injected into the “bubbles”, re-accelerated by **turbulence**, synchrotron cool and escape
- **Gamma-ray emission origin:** *leptonic* **inverse Compton** (IC) scattering of Cosmic Microwave Background (CMB) and interstellar radiation field (ISRF) photons
- **X-ray emission origin:** synchrotron emission

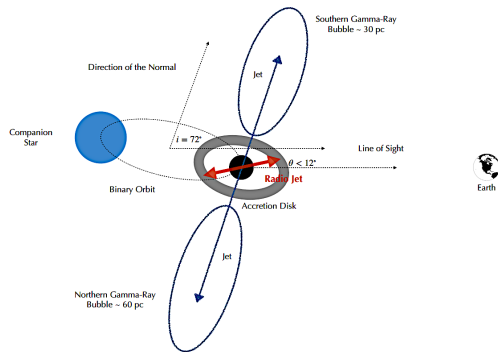


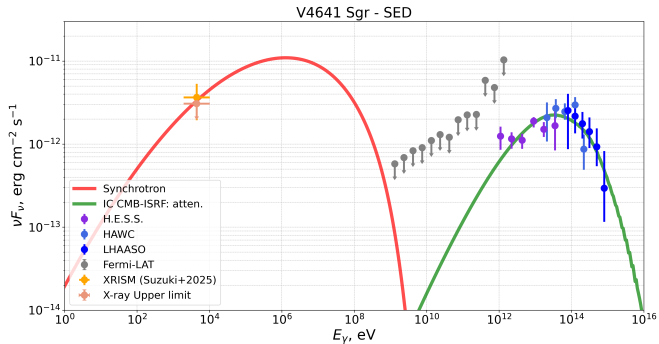
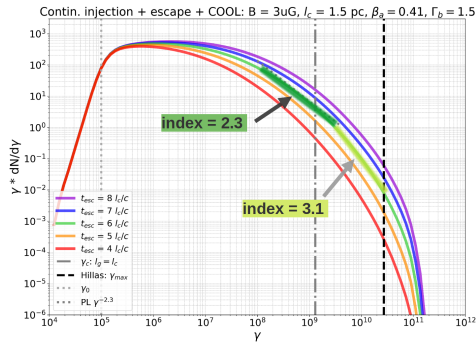
Figure: HAWC collaboration (2022)

V4641 Sgr: fitting SED data from H.E.S.S. & HAWC & LHAASO + XRISM

Physical parameters:

B (μG)	l_c (pc)	β_a	t_{esc} (l_c/c)	n_e (cm^{-3})	Γ_b	γ_0	$\delta B/B$	σ
3	1.5	0.41	6	5×10^{-14}	1.5	10^5	0.64	0.7

Results published in: Dmytriiev, van der Merwe, Böttcher (2025), arXiv:2512.02574, [PoS\(HEASA2025\)069](#)



Peculiar properties:

- Narrow SED bumps
- Synchrotron peaking in **X-ray** (1 – 10 keV)
- γ -ray bump peaking in 2 – 10 TeV
- **Hard X-ray/VHE spectra** ($\Gamma < 1.5 - 1.9$)
- *Hint of variability* ! (Goswami et al. (2024))

Models require artificial conditions:

- very high $\gamma_{\min}$
- extremely hard particle spectra

Particle acceleration mechanism ?

- Multiple successive shock acceleration ?
(Zech & Lemoine (2021))

Emission mechanism ? → SSC / lepto-hadronic ?

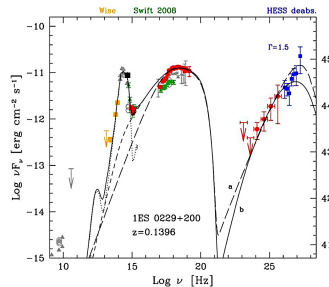
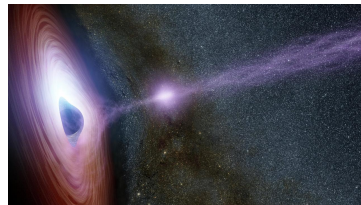


Figure: Modeling of the SED of 1ES 0229+200 with one-zone SSC scenario (Costamante et al. (2018))

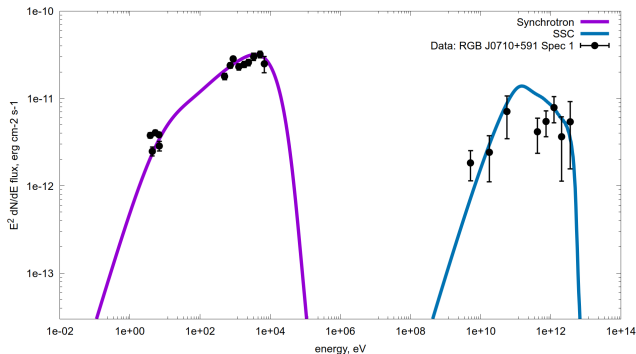
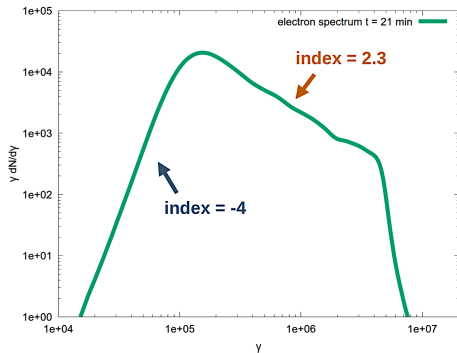
Extreme blazar fit example: RGB J0710+591 (high state)

Physical parameters:

PRELIMINARY

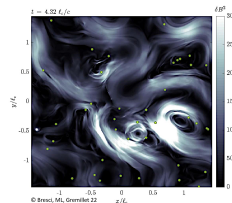
B (G)	l_c (cm)	β_a	n_e (cm $^{-3}$)	Γ_j	i_j (deg)	$t_{\text{acc,dur}}$ (h)
0.01	3.24×10^{14}	0.41	9×10^{-1}	10	16	0.35

Emission origin: SSC. **High state:** instantaneous injection of electrons with $\gamma_0 = 10^4$.
Evolve for $t_{\text{acc,dur}} = 21$ min (stochastic re-acceleration). l_c corresponds to $t_{\text{var}} \sim 1$ hour.



Summary and Outlook

- We developed a Monte-Carlo code STRIPE implementing a theoretical framework (Lemoine 2022) of stochastic particle acceleration by **high-amplitude relativistic turbulence with synchrotron/IC cooling**
 - The resulting spectra (instant. inj.) display **narrow peaks** and **extended power law tails**: *consequence of intermittency and high jumps in momentum*
 - **Higher turbulent Alfvén velocity β_a yields harder spectra.**
Weak dependence of spectral index on B and I_c
 - The spectrum (contin. inj.) is **curved**, with index α depending on t_{esc} , and can be very hard ($\alpha > 1$)
 - The acceleration model is able to reproduce hard TeV-to-PeV γ -ray spectrum of microquasar V4641 Sgr
 - The model also naturally explains peculiar properties of extreme blazar SEDs
-
- In future: more applications to various sources!



We'd like to thank Martin Lemoine and Luca Comisso for their valuable suggestions !

Thank you for your attention!

Ancient astronomers naming objects:



Yeah Jupiter is massive and vicious like
Zeus, let's name its moons
Io, Europa, Ganymede and Callisto
after lovers of Zeus.



Astronomers today:



PKS 1510-089
V4641 Sgr
RGB J0710+591
1ES 3599FDKGJSJSQQQQ